Isospin breaking in the reaction $np \to d\pi^0$ at threshold

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Abstract

The model for charge symmetry breaking in the reaction $np \to d\pi^0$ applied earlier around the Δ region is used to calculate the integrated forward-backward asymmetry of the cross section close to threshold. The mixing of the π and η mesons appears as strongly dominant at these energies. This contrasts elastic np scattering experiments, where the np mass difference in OPE dominates, or $np \to d\pi^0$ closer to the Δ region.

1 Introduction

Tests of charge symmetry and its breaking (CSB) in elastic two nucleon interaction fall into two categories according to how the investigated interaction behaves with respect to the total isospin. In the classification of Henley and Miller [1] a class III force depends on the zeroth component (in spherical tensor notation) of the total isospin as $\tau_{10} + \tau_{20}$ and is nonzero only for the pp and nn states (with opposite signs for the two). This has been investigated for decades in low-energy NN scattering and mirror nuclei with the main difficulties being extraction of the Coulomb force in the pp system and the lack of neutron targets in nn scattering [2, 3]. Clearly this interaction acts only in isospin one states and cannot change the value of the isospin. The same is true also for the isotensor force, class II. In contrast, a class IV force proportional to either $\tau_{10} - \tau_{20}$ or $(\vec{\tau}_1 \times \vec{\tau}_2)_0$ necessarily changes the isospin and therefore can act only in the np system, where both isospin zero and one are allowed.

The origin of class IV forces has three main sources, which are overall nearly equally important in elastic scattering:¹ i) the np-mass difference, ii) $\rho^0\omega$ -meson mixing and iii) the magnetic interaction of the neutron with the proton current. At the turn of the decade this isospin breaking interaction was seen in experiments as a difference of the neutron and proton analyzing powers $\Delta A = A_n - A_p$ in polarized np scattering [4, 5, 6].

¹Experiments are performed at a single angle where the np-mass difference in OPE dominates.

The class IV force can also show up in an interesting way in inelasticities. Namely, isospin respecting mechanisms in $NN \to d\pi$ involve only isospin one initial states. This sets strict constraints to the spins and parities of the initial states relative to the angular momentum of the final state pion: for odd l_{π} only singlet-even initial states are possible and for even l_{π} only triplet-odd. The separation of initial spins for different parities leads to a symmetric unpolarized cross section as stipulated by the Barshay-Temmer theorem for pure isospin reactions [7]. Obviously the presence of a class IV force can mix some isospin zero component in the initial state with opposite spin-parity assignments: Initial spin states will then have both parities involved. Consequently the cross section is no more exactly symmetric about 90° [8]. Presently there is an on-going experiment at TRIUMF [9] attempting to measure this asymmetry in the reaction $np \to d\pi^0$.

In Ref. [10] a few CSB mechanisms were studied in this reaction above 350 MeV. As a class IV force behaves spatially much like the spin-orbit force, in low energy scattering its effects vanish. In $np \to d\pi^0$ one can also argue for the smallness of asymmetry at threshold, because it must be an interference of opposite parity amplitudes, at least s- and p-wave pions, whereas the symmetric cross section has a squared s-wave term. Therefore, understandably Ref. [10] found quite small asymmetries at and below 400 MeV and did not aim to any details in the threshold region. However, it turns out that there are experimental advantages at threshold allowing smaller relative asymmetries to be detected than at higher energies, even at the level of one part in a thousand. The TRIUMF experiment E704 [9] utilizes this feature to measure the integrated asymmetry close to threshold. Also, theoretically at threshold there seems to be less cancellation of possible $\eta\pi$ mixing effects than at higher energies studied in Ref. [10]. This paper aims now to provide some detailed predictions for this observable in the threshold region where the experiment is performed.

2 Theory

2.1 CSB mechanisms

A standard source of the class IV force, dominant in most experiments, is the np mass difference in pion exchange. Taking this into account the pion-nucleon coupling becomes

$$H_{\pi NN} = -\frac{f}{\mu} \left[\vec{\sigma} \cdot \nabla \vec{\phi} \cdot \vec{\tau} + \delta \vec{\sigma} \cdot \nabla \phi_0 + \delta \vec{\sigma} \cdot (\vec{p} + \vec{p}') (\vec{\tau} \times \vec{\phi})_0 \right]$$
 (1)

with the small parameter $\delta = (M_n - M_p)/(M_n + M_p) = (M_n - M_p)/2M$. Here the first term is the familiar isospin invariant interaction and gives rise to the well known OPE potential. The initial and final momenta \vec{p} and \vec{p}' operate on the nucleons and ∇

operates on the pion field assumed a plane wave. The latter two terms give rise to the CSB potential of the form (using the usual notations of the literature [10])

$$V_{\delta} = \delta \frac{f^2}{4\pi} \frac{\mu}{3} \left\{ (\vec{\tau}_1 + \vec{\tau}_2)_0 \left[S_{12} V_T(\mu r) + \vec{\sigma}_1 \cdot \vec{\sigma}_2 V_C(\mu r) \right] - 6(\vec{\tau}_1 \times \vec{\tau}_2)_0 \left(\vec{\sigma}_1 \times \vec{\sigma}_2 \right) \cdot \vec{L} V_{LS}(\mu r) \right\}. \tag{2}$$

Here the first part is of class III, conserves the isospin and also with respect to the space and spin acts similarly to the normal OPE (i.e. has the standard tensor and spin-spin parts). However, the latter term changes both the spin and isospin being of class IV, i.e. couples the two possible spins for a given L = J partial wave. The associated set of coupled Schrödinger equations is solved numerically.

In pion physics, production, scattering as well as in the two-nucleon interaction, the coupling of the pion and nucleon to the πN resonance $\Delta(1232)$ is extremely important and dominates some processes [11, 12]. An isospin breaking effect can arise also here from the mass differences between the neutron and proton as well as between different charge states of the Δ . For charged pions this gives an isovector correction to the standard coupling as follows [10]

$$H_{\pi\Delta N} = -\frac{f^*}{\mu} \left[\vec{S} \cdot \nabla \vec{\phi} \cdot \vec{T} + \delta \vec{S} \cdot (\vec{p} + \vec{p}') (\vec{T} \times \vec{\phi})_0 \right]. \tag{3}$$

Here the transition spin (isospin) operator \vec{S} (\vec{T}) changes the spin (isospin) $\frac{1}{2}$ particles to those with $\frac{3}{2}$. The mass difference between consecutive charge states of the Δ has been assumed to be the same as for the nucleons. In the coupling (3) there is no term corresponding to the middle term in Eq. (1). As above for the nucleons one gets an isospin symmetry breaking transition potential

$$V_{\delta}^{tr}(\text{OPE}) = \frac{\delta f f^*}{4\pi} \frac{\mu}{3} \left\{ T_{10} \left[S_{12}^{\text{II}} V_T(\mu r) + \vec{S}_1 \cdot \vec{\sigma}_2 V_C(\mu r) \right] - 6(\vec{T}_1 \times \vec{\tau}_2)_0 \left(\vec{S}_1 \times \vec{\sigma}_2 \right) \cdot \vec{L} V_{LS}(\mu r) \right\} + (1 \leftrightarrow 2).$$
(4)

In the tensor operator S_{12}^{II} now one spin operator has been replaced by the corresponding transition spin operator. All terms arise analogously to the NN case, the first terms from CSB at the nucleon vertex.

A notable feature in this isospin breaking transition potential is that, contrary to the case of the NN interaction, also the first term in (4) (analogous to the class III term) can cause a transition from an isospin zero NN state to an intermediate ΔN state which can directly participate in pion production.

In addition to the possibility of the isospin mixing in the initial np state, the pion coupling (1) gives a possibility also for isospin breaking in the vertex generating the final pion state. From the baryon point of view the middle term is like a coupling of an isoscalar

meson, since there is no isospin operator. This means that there is a finite amplitude of direct transition from an initial isospin zero state to the deuteron state (and the pion). Of course, there is the same small parameter δ associated as in the isospin breaking potentials.

There are other possible isospin breaking mechanisms. Analogously with the above effective isoscalar meson coupling, also production of first a true off-shell isoscalar pseudoscalar meson (η or η') is possible with its subsequent transformation into pion, because there is a nonvanishing mixing between the η and π mesons [13, 14]. The coupling of pions to nucleons via this is of the form

$$H_{\eta\pi}^{\text{prod}} = -\frac{f_{\eta}}{\mu} \frac{\langle \eta | H | \pi \rangle}{\mu^2 - \eta^2} \vec{\sigma} \cdot \nabla \phi_0$$
 (5)

taking into account the two time orderings of the production and mixing interactions. (Ref. [10] had only the main one.) Using the mixing matrix $\langle \eta | H | \pi \rangle = -5900 \,\text{MeV}^2$ [14] and the ηNN coupling $G_{\eta}^2/4\pi = 3.68$ [15] with $f_{\eta} = G_{\eta}\mu/2M$ it can easily be seen that the strength of this contribution should be about 15 times larger than the isoscalar meson like coupling of the pion from the np mass difference in Eq. (1). So one would expect this to be a very important effect. This is further enhanced by the η' meson mixing with the mixing matrix element $-5500 \,\text{MeV}^2$ [14]. (The coupling of the η' to the nucleon is taken to be the same.)

There are great uncertainties in the ηNN and $\eta' NN$ coupling strengths G_{η} . Much smaller values are also quoted from pion photoproduction [16] and a sensitive probe for this coupling is desirable to clarify the situation. The above value is obtained in a meson exchange NN potential model fit to elastic NN scattering and is consistent (in the upper end) with the range 2–7 given in various versions of the Bonn potentials [17]. The latter also include a form factor of the monopole form at the ηNN vertices with Λ =1500 MeV (one potential uses even 2000 MeV). For this particular part of isospin breaking the former value of the cutoff is adopted as well as the Bonn cut-off 1300 MeV is also used for the pion. It should be further noted that the above value quoted in Ref. [15] is given as the strength of the meson coupling, i.e. for $|\mathbf{k}| = 0$ rather than at the meson pole. So in the normal Bonn parametrization this would correspond to $G_{\eta}^2/4\pi = 4.8$, very close to the most common value 5 in the Bonn potentials.

Another uncertainty is related to a controversy of off-shell $\rho\omega$ -meson mixing. The mixing matrix for this is determined on-shell, since both mesons have basically the same mass. Arguments have risen, based on fermion loop calculations and QCD sum rules, that the effect of $\rho\omega$ mixing should be much smaller than previously estimated in class IV interactions, because of the alleged off-shell momentum dependence of the mixing matrix [18]. However, this is contested in a newer analysis of the mixing matrix [19].

The effect of this off-shell modification, if necessary, would be to make this contribution nearly negligible in the ΔA of np elastic scattering [20] causing trouble with the data at low energy [5]. A similar effect could affect also $\eta \pi$ mixing [21]. However, in this case one particle is always off-shell even in determinations of the mixing matrix, because the masses are so different.

In the NN sector $\eta\pi$ mixing causes only a class III force. As seen above for the pion even this kind of coupling can produce an $NN \to \Delta N$ transition potential

$$V_{\eta\pi}^{tr} = -\frac{f_{\pi}^{*} f_{\eta}}{4\pi} \frac{\mu}{3} \frac{\langle \eta | H | \pi \rangle}{(\eta^{2} - \mu^{2})} T_{10} \left\{ S_{12}^{II} \left[V_{T}(\mu r) - \left(\frac{\eta}{\mu} \right)^{3} V_{T}(\eta r) \right] + \vec{S}_{1} \cdot \vec{\sigma}_{2} \left[V_{C}(\mu r) - \left(\frac{\eta}{\mu} \right)^{3} V_{C}(\eta r) \right] \right\} + (1 \leftrightarrow 2),$$
(6)

which can act also in isospin zero initial states. This potential includes now all time orderings of the meson production, absorption and mixing interactions and looks like a pion exchange with an η -ranged cut-off. In this way, as a two-step process also $\eta \pi$ mixing can produce an isospin mixing effect even in np scattering [22]. Due to the rather strong effective coupling seen above, also this should have a significant effect in pion production. Of course, the pion and η form factors are included in the potential.

The isospin symmetric amplitudes are calculated in a standard way generating the important isobar configurations in the initial states by solving coupled Schrödinger equations (coupled also with isospin zero np states) as described e.g. in Ref. [23], the procedure dating back basically over two decades [24]. This accounts then for both the direct production mechanism and pion rescattering through the Δ . Also pion s-wave rescattering from the second nucleon is taken into account in production. Details of the present potentials can be found in Ref. [25]. They reproduce the height of the $pp \to d\pi^+$ well and NN phase shifts to an accuracy of a few degrees from threshold over the Δ region. Here the pion cut-off is softer, but this transition potential is in the present context chosen to give the right overall strength of the $NN \to \Delta N$ transition, whereas the CSB one has the previously described parametrization to be consistent with the sources of the ηNN coupling constant.

2.2 Amplitudes

The asymmetry of the unpolarized cross section arises because there are opposite parities interfering in the same spin states. Close to threshold it involves just an interference of s- and p-wave pions, mainly of the CSB amplitude ${}^{1}P_{1} \rightarrow {}^{3}P_{1} \rightarrow s$ with the important

 $^1D_2 \rightarrow ^5 S_2(\Delta N) \rightarrow p$, if CSB is constrained to the NN sector. Due to the P-wave in the initial states the CSB ΔN mixing is not expected to be very important. However, CSB p-wave pions can arise from $^3D_2 \rightarrow ^1 D_2 \rightarrow p$ or from the Δ excitation process $^3D_2 \rightarrow ^5 S_2(\Delta N) \rightarrow p$ interfering with the dominant $^3P_1 \rightarrow s$ amplitude. This process has the advantage of producing the ΔN intermediate state in a lower angular momentum state than the initial nucleons. This is similar to the isospin respecting $^1D_2 \rightarrow ^5 S_2(\Delta N) \rightarrow p$ process showing a resonant (and dominant) structure in the Δ region.

Due to angular momentum and parity conservation, the class IV interaction between nucleons can only connect singlet and triplet states with L=J, i.e. only tensor uncoupled states. However, in an expanded baryon space also tensor coupled isospin zero states can experience isospin breaking transition to ΔN intermediate states [26]. This brings in e.g. the transition chain $^3D_1 \to ^3S_1(\Delta N) \to p$ for p-wave pion production, analogous to the dominant $^1D_2 \to ^5S_2(\Delta N) \to p$ and possibly a resonant structure at the Δ energies. Also $^3S_1 \to ^3S_1(\Delta N) \to p$ is possible.

Overall, all the processes considered above lead to indistinguishable final states and must be added coherently in the amplitudes.

3 Results and conclusion

In Table 1 complex contributions to CSB partial wave amplitudes from various components already discussed above are presented separately at the laboratory energy 279.5 MeV. The np mass difference in pionic potentials acts mainly in the NN sector in this energy region. The class IV force has no effect in the tensor coupled np states as shown above, whereas isobar configuration mixing gives a small contribution also there. However, the effect of these configurations is suppressed by nearly an order of magnitude as compared with the NN contribution. The effects at this energy are nearly the same in s-wave and p-wave production (from 3D_2) even at this low energy. At higher energies p waves gain in importance in proportion to the pion momentum. Clearly at this energy the pionic d waves and higher can be omitted: they are suppressed by an order of magnitude.

The isoscalar meson coupling like production vertex (from the middle term in Eq. (1)) gives a contribution of the same order of magnitude, but appears with the same strength also in the tensor coupled 3S_1 and 3D_1 initial states. There its effect is much more than the coupled isospin breaking channels - nearly all comes from the isospin breaking in the production vertex.

As anticipated earlier, the $\eta\pi$ mixing in the production vertex is about twenty times stronger than for the pion. As the $\eta\pi$ mixing potential appears effectively only in the Δ chains with the pion and η cutting off each other, the relative role of the production

Table 1: Relative importance of contributions to CSB amplitudes at the laboratory energy 279.5 MeV from the isoscalar meson like pion production vertex, isospin symmetry breaking pionic NN potentials due to the np mass difference in the NN sector and in the extended two-baryon space, η and η' production followed by transformation into a π^0 and potentials involving $\eta\pi$ or $\eta'\pi$ mixing.

Amplitude	π vertex	π pot. (NN)	π pot. (tot.)	η vertex	$\eta\pi$ potential
$^{1}P_{1} \rightarrow s$	(-0.45, 0.24)	(0.25, 0.60)	(0.10, 0.66)	(-9.04, 4.78)	(-1.24, 0.10)
${}^3S_1 \to p$	(-0.21, -0.07)	0	(0.02, 0.01)	(-4.09, -1.39)	(0.42, 0.09)
$^3D_1 \to p$	(-0.35, 0.13)	0	(0.01, -0.00)	(-7.06, 2.54)	(0.14, -0.02)
$^3D_2 \to p$	(0.24, 0.10)	(0.05, -0.58)	(-0.04, -0.69)	(4.85, 2.07)	(-2.21, -1.29)
$^{1}P_{1} \rightarrow d$	(-0.02, 0.01)	(0.00, -0.01)	(0.01, -0.01)	(-0.32, 0.17)	(0.02, 0.00)
$^1F_3 \to d$	(-0.01, 0.00)	(0.00, -0.02)	(0.00, -0.02)	(-0.30, 0.00)	(0.07, 0.00)

vertex is even enhanced. In the threshold region this single mechanism rises above the others in importance. The isobar effect arising from $\eta\pi$ mixing is overall still stronger than the class IV mixing in the NN sector from OPE. In these comparisons the absolute normalization and dimension are immaterial, but in relating different J values it should be known that these are reduced matrix elements of the two-nucleon system in the sense of Ref. [27] with the 3j-normalization. Furthermore, it is of interest to note that at this energy the sizes of the isospin conserving s- and p-wave (from 1D_2) amplitudes just happen to be a hundred times those of the η -vertex column (p wave from 3D_2).

The quantity of experimental interest here is the integrated forward-backward asymmetry divided by the total reaction cross section

$$A_{fb} \equiv \int [\sigma(\theta) - \sigma(\pi - \theta)] d\Omega / \int \sigma(\theta) d\Omega.$$
 (7)

Here the angle is the CM angle between the deuteron and incident neutron directions. In Table 2 this is given in per cent (i.e. it is multiplied by 100) for a range of energies in the neighbourhood of threshold.

The third column shows the contribution from the CSB isoscalar pion coupling at the production vertex. This is rather a minor contribution and is, in fact, more than cancelled by isospin breaking pion potentials (columns 4 and 5), mainly OPE in the nucleon sector at these energies, but also some amount from transitions into ΔN intermediate states. Both of these are dwarfed by the effects of $\eta\pi$ mixing. The column denoted by " η vertex" gives again the contribution from the CSB production vertex: a generated off-shell η meson changes into the final state on-shell pion. This is the dominant contribution but not at all as massively as expected from the sizes of the amplitudes. Comparably important

Table 2: Contributions to the integrated forward-backward asymmetry (%) from the isoscalar meson like pion production vertex, isospin symmetry breaking pionic NN potentials in the NN sector and in the extended two-baryon space, η and η' production followed by transformation into a π^0 and potentials involving $\eta\pi$ or $\eta'\pi$ mixing. The total should be the sum of π vertex, π pot. (tot.), η vertex and $\eta\pi$ potential. For comparison, the anticipated precision of the experiment E704 [9] is 0.12 %.

$E_{\rm lab}~({\rm MeV})$	$\eta = q/m_\pi^0$	π vertex	π pot. (NN)	π potential	η vertex	$\eta\pi$ potential
278	0.138	-0.008	0.026	0.020	-0.156	-0.117
279.5	0.170	-0.009	0.032	0.024	-0.183	-0.141
281	0.197	-0.010	0.036	0.028	-0.205	-0.161
285	0.255	-0.012	0.045	0.035	-0.247	-0.201
290	0.314	-0.014	0.054	0.041	-0.276	-0.236
300	0.408	-0.015	0.064	0.049	-0.292	-0.283
320	0.555	-0.013	0.073	0.056	-0.260	-0.337
350	0.730	-0.009	0.077	0.057	-0.176	-0.382

comes here the isospin breaking $\eta\pi$ transition potential and the importance of the latter increases with increasing energy. In the Δ region it is the largest individual contribution [10]. This happens, because both production vertex effects turn back down at the highest energies shown. In fact, in the neighbourhood of $E_{\rm lab} \approx 400$ MeV they even change sign leading to significant cancellation of the $\eta\pi$ mixing effects in the Δ region as was found in Ref. [10].

Preliminary calculations indicate that also the ρ and $\rho\omega$ -mixing effects as well as the electromagnetic interaction are significantly smaller than $\eta\pi$ mixing. These are in the same order as the pion effects. Therefore, as a summary it seems that CSB threshold production is strongly dominated by $\eta\pi$ mixing suggesting CSB measurements as an effective tool to study this phenomenon and constrain the ηNN coupling.

References

- [1] E. M. Henley and G. A. Miller, in "Mesons in Nuclei", eds. M. Rho and D. H. Wilkinson (North Holland, Amsterdam, 1979) p. 405.
- [2] G. A. Miller, B. M. K. Nefkens and I. Slaus, Phys. Rep. C **194** (1990) 1.
- [3] G.A. Miller and W.T.H. Van Oers, in *Fundamental Symmetries*, edited by W.C. Haxton and E.M. Henley (World Scientific, Singapore, 1995), p 127.

- [4] R. Abegg et al., Phys. Rev. D 39 (1989) 2464.
- [5] L.D. Knutson et al., Nucl. Phys. A 508 (1990) 185.
- [6] J. Zhao et al., Phys. Rev. C 57 (1998) 2126
- [7] S. Barshay and G. M. Temmer, Phys. Rev. Lett. 12 (1964) 728.
- [8] C.-Y. Cheung, E. M. Henley and G. A. Miller, Nucl. Phys. A 305 (1978) 342; A 348 (1980) 365.
- [9] TRIUMF experiment E704, spokespersons A. K. Opper and E. Korkmaz.
- [10] J. A. Niskanen, M. Sebestyen and A. W. Thomas, Phys. Rev. C 38 (1988) 838.
- [11] H. Garcilazo and T. Mizutani, πN Systems (World Scientific, Singapore 1990).
- [12] P. Wilhelm and H. Arenhövel, Nucl. Phys. A 593 (1995) 435.
- [13] P. C. McNamee et al., Nucl. Phys. A 249 (1975) 483; S. A. Coon, et al., Nucl. Phys. A 287 (1977) 381.
- [14] S. A. Coon, et al., Phys. Rev. D 34 (1986) 2784.
- [15] O. Dumbrajs et al., Nucl. Phys. B 216 (1983) 277.
- [16] L. Tiator et al., Nucl. Phys. A **580** (1994) 455.
- [17] C. Elster, R. Machleidt and K. Holinde, Phys. Rep. 149 (1987) 1; R. Machleidt, Adv. Nucl. Phys. 19 (1989) 189.
- [18] T. Goldman, J. A. Henderson and A. W. Thomas, Few Body Problems 12 (1993)
 123; J. Piekarewicz and A. G. Williams, Phys. Rev. C 47 (1993) 2462; T. Hatsuda,
 E. M. Henley, T. Meissner and G. Krein, Phys. Rev. C 49 (1994) 452.
- [19] S. A. Coon and M. D. Scadron, Phys. Rev. C 51 (1995) 2923.
- [20] M. J. Iqbal and J. A. Niskanen, Phys. Lett. B 322 (1994) 7.
- [21] J. Piekarewicz, Phys. Rev. C 48 (1993) 1555; K. Maltman and T. Goldman, Nucl. Phys. A572 (1994) 682.
- [22] J. A. Niskanen and A. W. Thomas, Aust. J. Phys. 41 (1988) 31.
- [23] J. A. Niskanen, Nucl. Phys. A 298 (1978) 417.

- [24] H. Arenhövel, Nucl. Phys. A **247** (1975) 473.
- [25] J. A. Niskanen, Phys. Rev. C **45** (1992) 2648.
- [26] J. A. Niskanen and A. W. Thomas, Phys. Rev. C 37 (1988) 1755.
- [27] A. de-Shalit and I. Talmi, Nuclear shell theory (Academic Press, NY, 1969).